A CARTOGRAPHIC APPROACH TO VENUS WIND MEASUREMENTS

Introduction

Carlos Romero and Jonathan Nickerson have both proposed methodologies that provide estimates of the arc length ρ between pairs of (x,y) measurements taken from Venus images. In addition, Jonathan's method calculates spherical coordinates (φ,λ) of each point relative to a rotated image of Venus; the origin of coordinates $(0,0)$ is the center of the circular image where $R = 1$. His computational method uses *haversine* functions to minimize large errors for small angles.

I propose an additional step that complements either of the two methodologies. This is a method that directly results in angular coordinates that are equivalent to latitude and longitude of a geographical coordinate system. The method relies on the fact that the Venusian *graticule* (the geographical coordinate grid) has properties very nearly (but not exactly) congruent with the common *orthographic map projection*. One important result is that measurements of point features are automatically converted into angular coordinates ϕ and λ (equivalent to latitude and longitude) of a coordinate system of an equatorial orthographic projection whose cartesian origin $(0,0)$ is the intersection of the Equator ($\phi = 0°$) and the Prime Meridian ($\lambda = 0°$). Use of a Venus ephemeris to provide the Venusian origin transforms the equatorial graticule into an oblique orthographic graticule; thus, (x,y) measurements of each point feature are transformed into true geographical coordinates.

I pursued this approach because of an intriguing possibility that surface features on Venus or in the lower atmosphere may influence circulation in the atmosphere above.

A Short Review of Geographic Projections

Despite the fact that visually Venus is clouded with a seemingly opaque atmosphere, features on the solid surface are located with respect to a coordinate grid (called a *graticule*) similar to Earth's geographic grid. The graticule comprises *great circles* (meridians designating longitude passing through the North and South poles of rotation) and *small circles* (parallels of latitude) designating angular distances from the Equator. The Equator, largest of the small circles and 90° distant from each pole, is also a great circle. As on Earth, the origin of coordinates of the Venus graticule is the intersection of the Equator ($\phi = 0°$) with the Prime Meridian ($\lambda = 0°$).

Transient features in the atmosphere are described with three coordinates: Latitude, longitude, and elevation above a standard datum (such as the mean radius R of the planet).

Assumptions

This paper describes a first step in establishing a coordinate grid appropriate for the Venus Winds project, from principles that were worked out over two millennia ago. For

simplicity, it is assumed that Venus is a regular spheroid of mean radius 6052 km. Furthermore, it is assumed that, as imaged from Earth, Venus shows exactly one-half of its surface at any instant; in effect, images of Venus are viewed as if the planet were located at infinity. These assumptions suggest that a map projection be used that depicts an accurate representation of cloud features in the images. A projection fitting this requirement is the *orthographic* projection, a venerable map projection that supposedly dates back to Hipparchos of Greece in the 2nd century BCE. The *oblique* orthographic projection (Fig. 1) is globelike and is often used in popular magazines and books to portray the relationship of geographic features as if viewed from deep space.

Another assumption is that the *initial* view direction (called the *equatorial aspect*) is normal to the Equator along a *central meridian* that appears as a straight line between the North and South poles (Fig. 2). While the center of the projection is also a point on the Equator, any longitude is possible. All parallels in the equatorial orthographic projection are straight lines and all meridians are segments of circles. The outer meridians, 90° to either side of the central meridian, make a complete circle. The equations for converting cartesian coordinates x_i,y_i into projected coordinates and the corresponding inverse formulae are found in Appendix A.

Fig. 1 Oblique orthographic projection (Snyder, 1987)

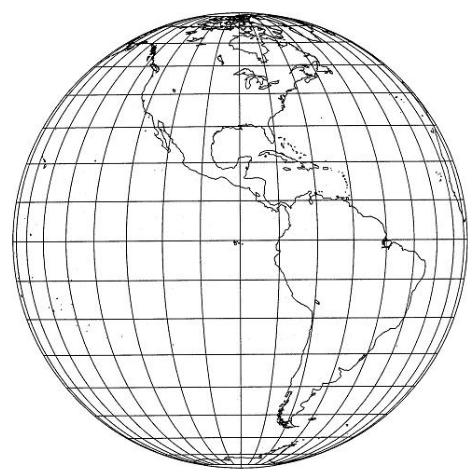


Fig. 2 Equatorial orthographic projection (Snyder, 1987)

Application to the Venus Winds Project

Consider the following application of the orthographic map projection methodology to the Venus Winds Project. I suggest that the map projection process is one method of analyzing the measured image data that adds significant value, namely, that those data contain geographic meaning (latitude and longitude) referenced to the actual Venusian coordinate system (similar to Fig. 1) and are obtainable with some additional processing.

First, some assumptions about the measurement process: Images of Venus are examined by analysts and coordinates x_i,y_i are measured with respect to a circumscribed circle (the *limb arc*) of radius R corresponding to the limb of the planet. For purposes of argument, assume that $R = 1.0$ unit and that the center of the circle is at $0,0$, (that is, the values of x_i,y_i are normalized within the circle to range from -1.0 to $+1.0$). An additional

registration process may also have rotated the image so that $x = 0$ intersects the limb arc at the poles. A temporary graticule is created to be precisely that of the equatorial orthographic projection (similar to Fig. 2). It is assumed that the axial rotation of the planet is negligible between measurements.

The following steps are for a sequence of measurements of a *single* cloud feature:

1. A set of sequential image coordinates of the cloud feature are measured from a series of images.
2. The series of coordinate pairs will be converted temporarily into coordinates of an equatorial orthographic projection using equations (7-10) that define the latitude ϕ_i and longitude λ_i of each point.
3. Choose a reference point in the series to be at the center of projection. Compute the arc distances ρ_i between it and the other points using equation (1). The reference point will usually be an end point in the sequence but could, in principal, be any point in the series.
4. Compute average velocity from the distance (in m) and elapsed time between each point and the reference point.

It is clear that arc length is easily computed using equations (7-10). If the run of excellent observations x_i,y_i where $i= 1,n$ and (ϕ_i,λ_i) are the coordinates of each point in the series, the average great circle arc c_i of any pair is obtained from equation (1).

In the future, if it is decided that the *actual* Venusian coordinates are important, the temporary coordinates described above could be easily rectified from equatorial orthographic to true Venusian coordinates using equations (7-10), by using an ephemeris to set the origin of coordinates (ϕ_1 and λ_0) for each series. In this instance, the potential for error assuming that the rotation of Venus is negligible is now eliminated.

APPENDIX A
Formulae Used on the Sphere

For a spherical triangle (Fig. 3) having angles *A*, *B*, and *C* and great circle arcs *a*, *b*, and *c* connecting them, and if *C* is located at the North Pole, *C* is the angle between two meridians extending through *A* and *B*.

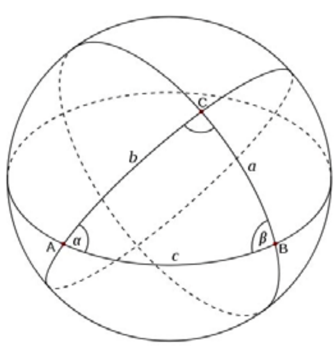


Fig. 3 Spherical triangle composed of sides *a*, *b*, and *c* and angles *A*, *B*, and *C*.

The notation of Snyder (1987) is used here. By the Law of Cosines, the great circle *c* connecting the center of projection *A* (ϕ_1, λ_0) and the point *B* (ϕ, λ) is given by

$$\cos c = \sin \phi_1 \bullet \sin \phi + \cos \phi_1 \bullet \cos \phi \bullet \cos (\lambda - \lambda_0) \tag{1}$$

The term *cos c* must be positive and less than or equal to 1.0; a value larger than 1.0 is beyond the outer limit of the projection.

$$\rho = R \sin c \tag{2}$$
$$\theta = \pi - Az = 180° - Az \tag{3}$$
$$x = R \cos \phi \bullet \sin (\lambda - \lambda_0) \tag{4}$$
$$y = R [\cos \phi_1 \bullet \sin \phi - \sin \phi \bullet \cos \phi \bullet \cos (\lambda - \lambda_0)] \tag{5}$$
$$h' = \cos c = \sin \phi_1 \bullet \sin \phi + \cos \phi_1 \bullet \cos \phi \bullet \cos (\lambda - \lambda_0) \tag{6}$$
$$k' = 1.0$$

where

ϕ_1 and λ_0 are the latitude and longitude of the center point *A*
ϕ and λ are the latitude and longitude of the point *B*
c = Angular distance of a given point from the center of projection
R = Radius of sphere
x,y = Projected coordinates of the point at *B*
ρ = Arc length between *A* and *B*
Az = Azimuth of ρ measured clockwise from *y*-axis
h' is the scale factor along a line radiating from the center of projection
k' is the scale factor in a direction perpendicular to a line radiating from the center

The inverse formulae are:

$$\rho = (x^2 + y^2)^{1/2}$$
(7)
$$c = \arcsin (\rho/R) \tag{8}$$
$$\phi = \arcsin [\cos c \bullet \sin \phi_1 + (y \sin c \bullet \cos (\phi_1 /\rho))] \tag{9}$$

If $\phi_1 = 90, \lambda = \lambda_0 + \arctan(x/(-y))$
If $\phi_1 = -90, \lambda = \lambda_0 + \arctan(x/y)$
Otherwise,

$$\lambda = \lambda_0 + \arctan [x \sin c/(\rho \cos \phi_1 \bullet \cos c - y \sin \phi_1 \bullet \sin c)] \tag{10}$$

For the equatorial orthographic projection

$$\phi_1 = 0 \tag{11}$$
$$\lambda_0 = 0 \tag{12}$$
$$x = R \bullet \cos \phi \bullet \sin (\lambda) \tag{13}$$

$$y = R \bullet \sin \phi \tag{14}$$

and the inverse formulae

$$\phi = \arcsin (y/R) \tag{15}$$
$$\lambda = \arcsin (x/R \cos \phi) \tag{16}$$

APPENDIX B
Worked Examples

Example #1 (after Snyder, 1987, p. 311)

This is a worked example of determining the *x,y* coordinates of an arbitrary point using the oblique orthographic projection.

Radius: R = 1.0
Center: $\phi_1 = 40° N., \lambda_0 = 100° W.$
Point: $\phi = 30° N., \lambda = 110° W.$

Find: x,y

First, determine if the point is within view by computing *cos c* from (1):

$$\cos c = \sin 40° \bullet \sin 30° + \cos 40° \bullet \cos 30° \bullet \cos (-110° + 100°) = 0.97$$

This is less than 1.0, so the point is within view.

Using equations (4) and (5),

$$x = 1.0 \cos 30° \sin (-110° + 100°) = -0.15$$
$$y = 1.0 [\cos 40° \bullet \sin 30° - \sin 40° \bullet \cos 30° \bullet \cos (-110° + 100°) = -0.17$$

Example #2 (after Snyder, 1987, p. 312)

This is a worked example of determining the *map* coordinates of an arbitrary point having coordinates *x,y* using the oblique orthographic projection and inverse formulae. Using the values determined in Example #1:

Radius: $R = 1.0$
Center: $\phi_1 = 40° N., \lambda_0 = 100° W.$
Point: $x = -0.15, y = -0.17$
Find: ϕ,λ

Using equations (7) through (10),

$$\rho = [(-0.15)^2 + (-0.17)^2]^{1/2} = 0.22$$

$c = \arcsin(0.22/1.0) = 12.9°$

$\phi = \arcsin[\cos 12.9° \bullet \sin 40° + (-0.17 \sin 12.9° \cos 40°/0.22] = 30°$ *N. (rounded off)*

$\lambda = -100° + \arctan[-0.15 \sin 12.9°/(0.22 \bullet \cos 40° \bullet \cos 12.9° + 0.17 \bullet \sin 40° \bullet \sin 12.9°]$
$= -100° + \arctan[-0.033/0.191] = -100° + (-10.0°) = 110°$ *W. (rounded off)*

Since the denominator of the argument of *arctan* is positive, the adjustment for quadrant is not necessary.

APPENDIX C
Hypothetical Example

This example shows how conversion of measurements x_i,y_i into velocity values would be accomplished by referencing to a temporary graticule of an equatorial orthographic projection. Refer to Excel spreadsheet ORTHO3a.xls, which shows a hypothetical tabulation of pairs of x_i,y_i measurements of cloud features imaged at known time intervals.

Several important assumptions must be emphasized, namely, that during the observing run:

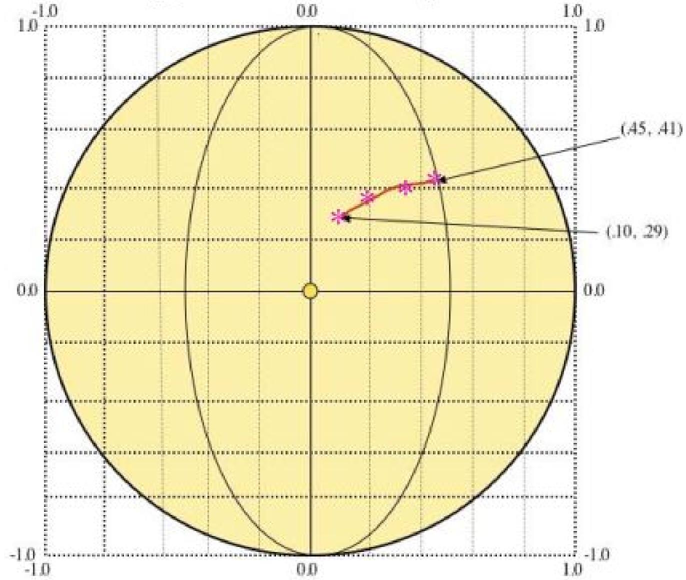
1. The scale factor (number of pixels for R = 1.0) remains reasonably constant
2. Rotation of the image plane is negligible.
3. Axial rotation of the planet is negligible

I believe that these conditions can be satisfied by careful registration of each image in advance of measuring the image coordinates of features.

DEFINITION OF COLUMNS:

PAIR#: Index number of measurement pairs
X1, Y1: First measurement coordinates normalized to the range for each is -1.0 to 1.0.
LAT1, LONG1: Computed equatorial orthographic map coordinates, in degrees
X2, Y2: Second measurement coordinates normalized to the range for each is -1.0 to 1.0.
LAT2, LONG2: Computed equatorial orthographic map coordinates, in degrees
COSC: Value of *cos c* as in equation (6*)*
C(DEG): Angular distance (degrees) between the two points
RHO(KM): Arc length (km) between the two points where R = 6052 km
DeltaT(min): Representative values of elapsed time (min) between measurements
V(m/sec): Average velocity (m/sec) of feature

Hypothetical Example



PAIR#	X1	Y1	LAT1	LONG1	X2	Y2	LAT2	LONG2	COSC	C(DEG)	RHO(KM)	DeltaT(min)	V(m/sec)
1,2	0.45	0.47	28.03	30.31	0.35	0.41	24.20	22.43	0.99	8.04	849.15	990.50	85.73
2,3	0.35	0.41	24.20	22.43	0.20	0.36	21.10	12.34	0.99	9.82	1036.67	1345.90	77.02
3,4	0.20	0.36	21.10	12.34	0.10	0.29	16.86	5.99	0.99	7.35	776.12	823.50	94.25
SUM 1-4										25.21	2661.94	3159.90	84.24
1,4	0.45	0.41	24.20	29.38	0.10	0.29	16.86	5.99	0.92	23.07	2436.08	3159.90	77.09

Fig. 4 Hypothetical example of translation of image coordinates to orthographic projection (equatorial aspect) coordinates

Note that a constant displacement of 0.1 unit in both *x* and *y* in the image plane corresponds to an arc length RHO that is numerically larger the farther the displacement is from the origin at *(0.0,0.0).* This effect is equivalent to foreshortening of an arc of constant length the farther the arc is from the center. Note also that the sum of the path lengths of the individual segments (PATH# = SUM 1-4) is longer than the great circle path (PATH# = SUM 1-4) requiring a faster average velocity to get from point #1 to point #4.

REFERENCE

Snyder, J.P., 1987, Map Projections--A Working Manual: U.S. Geological Survey Professional Paper 1395, 383 p.

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